A linear approximation for a stochastic optimal power flow problem based on wind energy sources

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Overview

1. Introduction
2. Mathematical models
3. Computational analysis
4. Conclusions

Introduction

Problem statement

The activities of management and planning of resources on a national scale of an electrical network involve solving several linear and nonlinear optimization problems. One of these problems is the Optimal Power Flow (OPF) problem.

In addition, integrating Renewable Energy Sources (RES) in a generation portfolio conveys several benefits, such as a reduction in greenhouse gas emissions and the country's dependency on imported energy, and it decreases spot prices. However, the generation from RES can be variable and uncertain, in contrast to conventional generation. Different OPF problems can be formulated in this stochastic setting.

Different approaches

Both linear[1] and nonlinear nonconvex[2] formulations can be found in the literature for the deterministic OPF problem.

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In addition, when dealing with uncertainties, both stochastic[3] and robust[4] optimization techniques are being used.

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We formulate this problem with a Stochastic Mixed Integer Programming (SMIP) model to solve a linear approximated formulation of a non-deterministic OPF.

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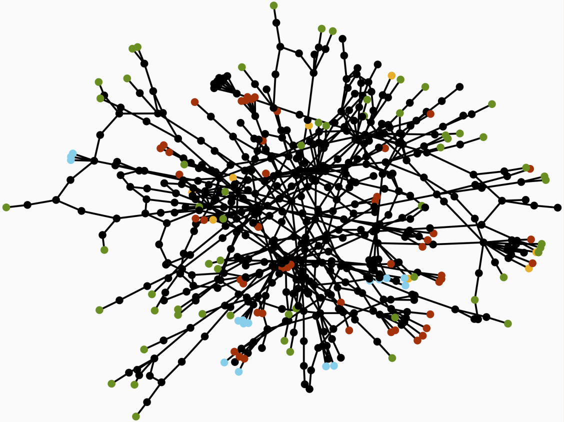
Goals

Our goal is to study an OPF problem regarding Sicily's electrical network.

At the moment, we are focusing on small subgraphs containing coal and wind generators only.



Generators: coal, solar, hydro, wind (CESI).

Mathematical models

Data and variables

Indices

i, j indices of buses;

k index for auxiliary variables.

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Sets

Ω_b set of all buses.

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Variables

m_{ijk} auxiliary binary variables;

$x_{ijk}, y_{ijk}, \alpha_{ij}, \beta_{ij}, \gamma_{ij}$ auxiliary continuous variables;

OC operation cost (€/h);

$\mathrm{PG}_i/\mathrm{QG}_i$ active/reactive power generation at bus i (MW/MVAr);

P_{ij}/G_{ij} active/reactive power flow of line ij (MW/MVAr);

$|V_i|$ voltage magnitude at bus i (kV);

θ_{ij} voltage angle difference between bus i and bus j (rad).

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Constants

a_i, b_i, c_i cost coefficients of active power generation at bus i (€/ MW^2h, €/ MWh, €/ h);

$\mathrm{B}M1_{ij}$, $\mathrm{B}M2_{ij}$ disjunctive parameters;

k_1 maximum value of k;

n constant for polyhedral approximation;

$\mathrm{PD}_i/\mathrm{QD}_i$ active/reactive power demand at bus i (MW/MVAr);

PG_i^{min}, QG_i^{min}, $|V_i|^{min}$, θ_{ij}^{min}, PG_i^{max}, QG_i^{max}, $|V_i|^{max}$,θ_{ij}^{max} minimun and maximum values of corresponding variables;

$|S_{ij}|^{max}$ maximum magnitude of apparent power of line ij (MVAr);

Y_{ij} admittance of line ij, $Y_{ij} = G_{ij} + jB_{ij}$ (℧);

θ_{ref} voltage phase angle for the slack bus ($\theta_{ref} = 0$) (rad);

$\Delta\theta_{ij}$ constant used in the linearization of the model (rad).

Deterministic model

$$
\begin{aligned}
\min \quad & \mathrm{OC} = \sum_{i \in \Omega_b} a_i \mathrm{PG}_i^2 + b_i \mathrm{PG}_i + c_i && && \text{(1a)}\\
\text{s.t.} \quad & \mathrm{PG}_i - \mathrm{PD}_i = \sum_{j \in \Omega_b} P_{ij} && \forall i \in \Omega_b, && \text{(1b)}\\
& \mathrm{QG}_i - \mathrm{QD}_i = \sum_{j \in \Omega_b} Q_{ij} && \forall i \in \Omega_b, && \text{(1c)}\\
& P_{ij} = |V_i|^2 G_{ij} - |V_i||V_j|(G_{ij} cos\theta_{ij} + B_{ij} sin\theta_{ij}) && \forall i, j \in \Omega_b, && \text{(1d)}\\
& Q_{ij} = -|V_i|^2 B_{ij} - |V_i||V_j|(G_{ij} sin\theta_{ij} - B_{ij} cos\theta_{ij}) && \forall i, j \in \Omega_b, && \text{(1e)}\\
& \mathrm{PG}_i^{min} \leq \mathrm{PG}_i \leq \mathrm{PG}_i^{max} && \forall i \in \Omega_b, && \text{(1f)}\\
& \mathrm{QG}_i^{min} \leq \mathrm{QG}_i \leq \mathrm{QG}_i^{max} && \forall i \in \Omega_b, && \text{(1g)}\\
& (P_{ij})^2 + (Q_{ij})^2 \leq (|S_{ij}|^{max})^2 && \forall i, j \in \Omega_b, && \text{(1h)}\\
& |V_i|^{min} \leq |V_i| \leq |V_i|^{max} && \forall i \in \Omega_b, && \text{(1i)}\\
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Linearization of constraints (1d) and (1e)

Starting from (1d), for every $i, j \in \Omega_b$ we can make the following approximation

$$P_{ij} \approx (2|V_i| - 1)G_{ij} - (|V_i| + |V_j| - 1)(G_{ij}(1 - \theta_{ij}^2/2) + B_{ij}\theta_{ij}). \tag{2}$$

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Defining $\gamma_{ij} = |V_i| + |V_j| - 1$, $\alpha_{ij} = \gamma_{ij}\theta_{ij}$, $\beta_{ij} = \gamma_{ij}\theta_{ij}^2$ we can rewrite (2) as

$$P_{ij} \approx G_{ij}(|V_i| - |V_j| + \beta_{ij}/2) - B_{ij}\alpha_{ij}. \tag{3}$$

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Now we just need to linearize γ_{ij}, α_{ij}, β_{ij}. First of all, we approximate the continuous decision variables θ_{ij} by a set of discrete variables as follows:

$$\theta_{ij} = \theta_{ij}^{min} + \Delta\theta_{ij} \sum_{k=0}^{k_1} 2^k m_{ijk}, \tag{4}$$

where $\Delta\theta_{ij} = (\theta_{ij}^{max} - \theta_{ij}^{min})/2^{k_1}$.

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where $\Delta\theta_{ij} = (\theta_{ij}^{max} - \theta_{ij}^{min})/2^{k_1}$. We then multiply by γ_{ij} and obtain

$$\alpha_{ij} = \gamma_{ij}\theta_{ij}^{min} + \Delta\theta_{ij} \sum_{k=0}^{k_1} 2^k x_{ijk}, \quad (5)$$

where $x_{ijk} = \gamma_{ij} m_{ijk}$.

Linearization of constraints (1d) and (1e)

We can then linearize $x_{ijk} = \gamma_{ij} m_{ijk}$ by

$$\begin{aligned} 0 \leq \gamma_{ij} - x_{ijk} \leq (1 - m_{ijk})\mathrm{B}M1_{ij}, \\ 0 \leq x_{ijk} \leq m_{ijk}\mathrm{B}M1_{ij}. \end{aligned} \tag{6}$$

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Finally, we can linearize β_{ij} with

$$\beta_{ij} = \gamma_{ij}\theta_{ij}^2 = \alpha_{ij}\theta_{ij} = \alpha_{ij}\theta_{ij}^{min} + \Delta\theta_{ij}\sum_{k=0}^{k_1} 2^k y_{ijk}, \tag{7}$$

where $y_{ijk} = \alpha_{ij} m_{ijk}$ can be linearized as

$$\begin{aligned} -(1 - m_{ijk})\mathrm{B}M2_{ij} \leq \alpha_{ij} - y_{ijk} \leq (1 - m_{ijk})\mathrm{B}M2_{ij}, \\ -m_{ijk}\mathrm{B}M2_{ij} \leq y_{ijk} \leq m_{ijk}\mathrm{B}M2_{ij}. \end{aligned} \tag{8}$$

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Constraint (1e) can be approximated in a similar way as

$$Q_{ij} \approx -B_{ij}(|V_i| - |V_j| + \beta_{ij}/2) - G_{ij}\alpha_{ij}. \tag{9}$$

Linearization of constraint (1h)

For every $i, j \in \Omega_b$, we start from

$$(P_{ij})^2 + (Q_{ij})^2 \leq (|S_{ij}|^{max})^2 \tag{1h}$$

and we use the following n linear approximations

$$\begin{aligned}
&(sin(2\pi l/n) - sin(2\pi(l-1)/n))P_{ij} \\
&- (cos(2\pi l/n) - cos(2\pi(l-1)/n))Q_{ij} \\
&- |S_{ij}|^{max} \times sin(2\pi/n) \leq 0
\end{aligned} \tag{10}$$

where $l = 1, \ldots, n$.

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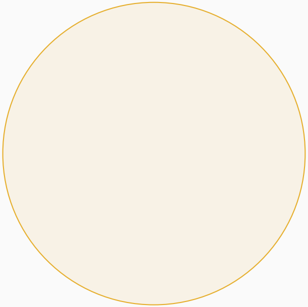

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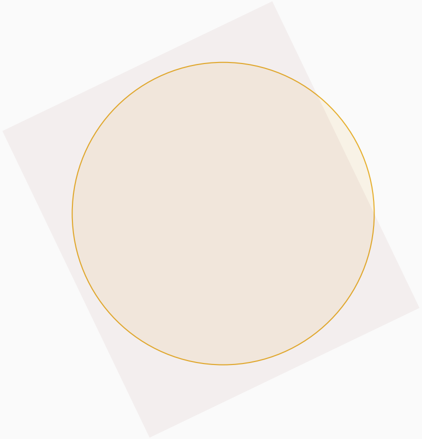

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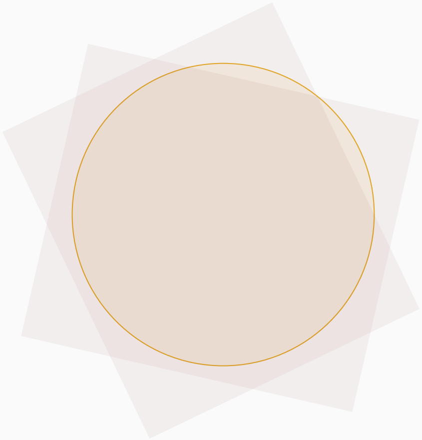

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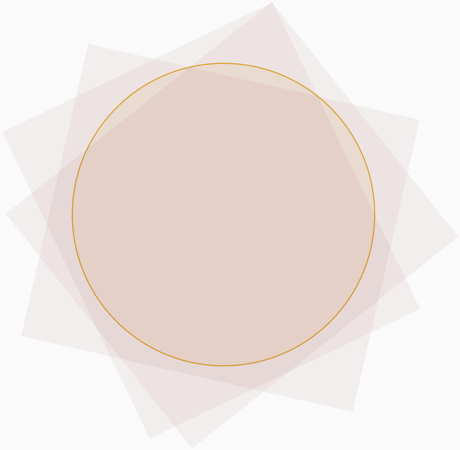

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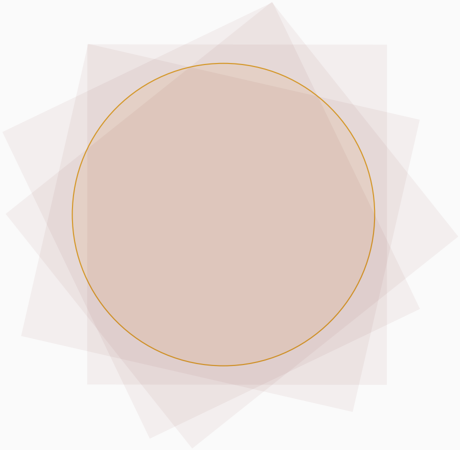

Linearization of constraint (1h)

For every $i, j \in \Omega_b$, we start from

$$(P_{ij})^2 + (Q_{ij})^2 \leq (|S_{ij}|^{max})^2 \tag{1h}$$

and we use the following n linear approximations

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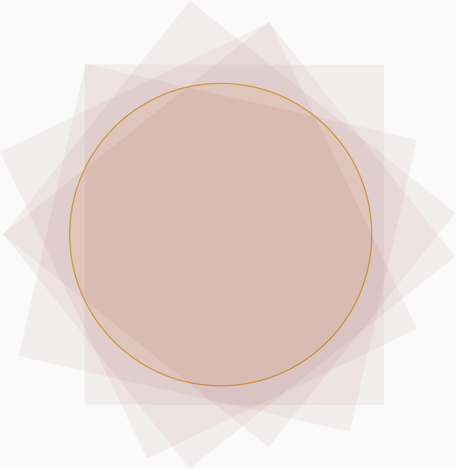

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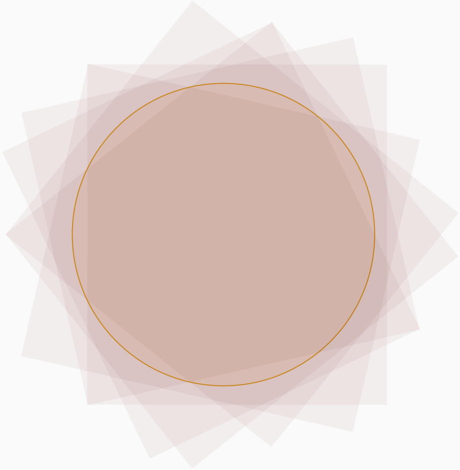

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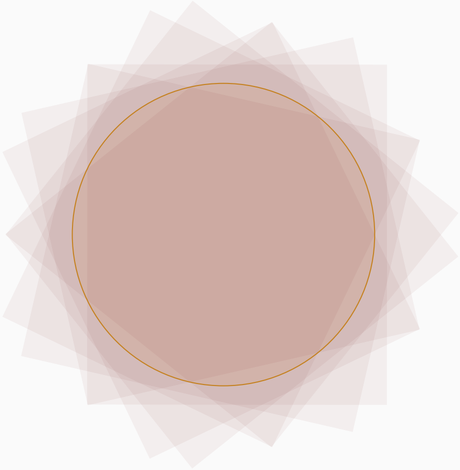

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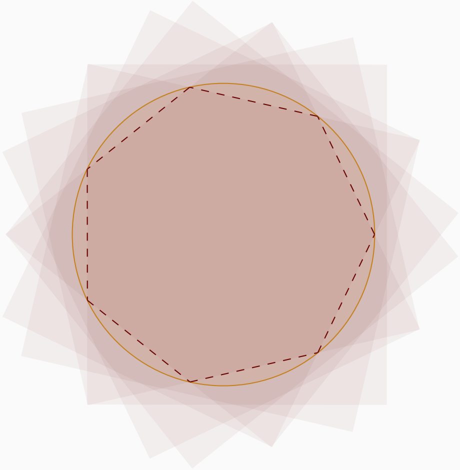
&- (cos(2\pi l/n) - cos(2\pi(l-1)/n))Q_{ij} \\
&- |S_{ij}|^{max} \times sin(2\pi/n) \leq 0
\end{aligned} \tag{10}$$

where $l = 1, \ldots, n$.



Adding stochasticity

When dealing with buses that have RES generators in addition to coal generators, power generation becomes a random variable. For example, regarding coal and wind generators, in every scenario $\xi \in \Xi$ we have that

$$\mathsf{PG}_{i\xi} = \mathsf{PG}^{c_i} + \mathsf{PG}^{w_i}_{\xi}. \tag{11}$$

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Constraint (1b) can be then formulated as a chance constraint

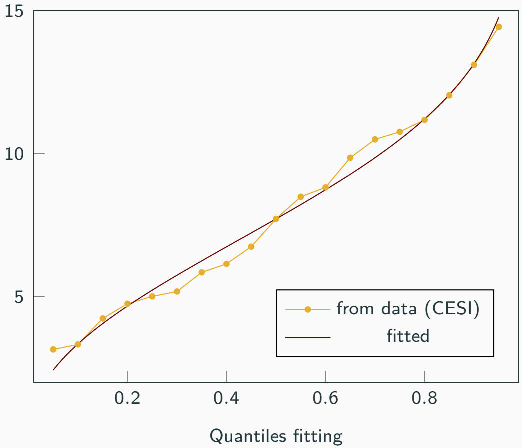
$$\mathbb{P}\left\{ \mathrm{PG}^{c_i} + \mathrm{PG}^{w_i} - \sum_{j\in\Omega_b} P_{ij} \geq \mathrm{PD}_i \right\} \geq 1-\beta, \quad \beta \in [0,1]. \tag{12}$$

We can model this constraint by fixing a discrete probability distribution using

- previous observations or
- samples drawn from a fitted probability distribution.

Computational analysis

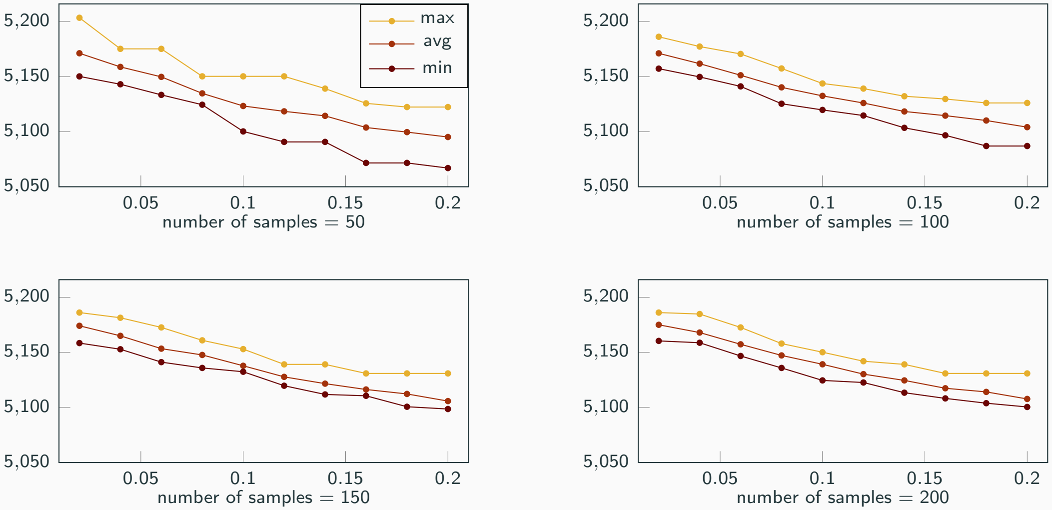
Scenario generation



Starting from a list of quantiles q_i of order α_i regarding wind power generation, we fitted a Weibull probability distribution of parameters $(\lambda,\, \kappa)$ in order to minimize

$$f(\lambda, \kappa) = \sum_{i=1}^{n} \left(q_i - \lambda \left(\ln \frac{1}{1-\alpha_i} \right)^{\frac{1}{\kappa}} \right)^2 \qquad (13)$$

Objective function with respect to β - 9 buses



Conclusions

Final remarks and future perspectives

Linear approximated OPF model in a deterministic and stochastic setting;

scenario generation based on a Weibull quantiles fitting;

changes in operation costs w.r.t different confidence levels.

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Linear approximated OPF model in a deterministic and stochastic setting;

scenario generation based on a Weibull quantiles fitting;

changes in operation costs w.r.t different confidence levels.

Test the model on a bigger graph and with different RES generators;

improve the linearization in order to relax the conditions on $|V_i|$ and θ_{ij};

formulate a data-driven distributionally robust approach.

Fine.